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ABSTRACT

Distinguishing two forms of inference (additive and general vicariance), this paper describes a procedure for assessing class vicariance that attempts to control potentially confounding effects of verbal comprehension. The term "class vicariance" refers to the inference that a class remains invariant under arbitrary dichotomous partitionings. Additive vicariance refers to inferences in which subclass structure is additive and involves no class multiplication. General vicariance refers to inferences in which classes are partitioned multiplicatively. A broader theoretical issue addressed by this research is that of whether additive and multiplicative operations on classes develop simultaneously or asynchronously. It is hypothesized that applying vicariance in the case of additive subclasses would be achieved earlier than applying it in the general case of multiplicative subclasses. A total of 72 children were tested individually on four additive and three general vicariance problems. Subjects ranged in age from 5 years to 7 years, 10 months. Findings support Piaget's interpretation of additive class vicariance as a distinct acquisition of the concrete operational period. Performance on these problems shows progressive development across the age range of 5 through 7 years. Clearly, though, children do not achieve consistent success on these problems even at age 7, and this is especially true for general vicariance problems. (RH)

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Development of the Class Vicariance Inference

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The term class vicariance refers to the inference that a class remains invariant under arbitrary dichotomous partitionings. Taking the class "humans", for example, vicariance permits infererces of the form, "the Chinese together with the non-Chinese are equal to the French together with the non-French."

Piaget formalized this inference as one of the eight concrete operational groupings (Grouping II), and speculated about its role in various achievements of the concrete operational period. For instance, the idea that a class can be divided into subclasses in a variety of ways may contribute to children's construction of class multiplication schemes (Piaget, 1972). The vicariance concept was also proposed as one of the logical prerequisites for quantification; thus, in conservation tasks, children must understand that the whole remains invariant under arbitrary rearrangements of its parts. Piaget (1941/1965) related logical vicariance specifically to number concepts, such as the notion that each natural number is decomposable in a variety of ways (e.g., 4 + 4 = 7 + 1); and Piaget (1942) presented a formal derivation of number directly from the vicariance grouping.

More recently, interest in vicariance has been revived in the context of commutability theory (Easley, 1978; Inhelder et al., 1975; Piaget, 1974/1980, Chap. 11; Pinard, 1981). Commutability is the inference that the order of arrangement doesn't affect quantity. This involves noticing that during a transformation, such as rearrangement of an array in a number conservation problem, the part added in one dimension or direction corresponds to the part removed in the reciprocal dimension or inverse direction. Thus, commutability is essentially a quantitative version of qualitative class vicariance.

In spite of its potential theoretical importance, development of class vicariance has seldom been investigated directly. Most of the evidence that children acquire the concept during the concrete operational period has been based on a rational analysis of the reasoning that goes into solving problems (such as conservation), which are not explicit measures of vicariance.

Research on vicariance has undoubtedly been hampered by the awkwardness of asking children questions such as, "are the Chinese plus the non-Chinese equal to the French plus the non-French?" Researchers may justly be skeptical about investigating children's logical competencies by means of problems that appear to entail serious issues of language comprehension. This paper describes a procedure for assessing class vicariance that attempts to control for verbal comprehension. Secondly, two forms of inference are distinguished: additive vicariance and general vicariance.

By Piaget's (1972) definition, the vicariance operation is defined on classes that may be partitioned into disjoint, or nonintersecting, subclasses. For instance, in the example using nationalities, it is assumed that no persons are both French and Chinese. By additive vicariance, I refer to inferences applied to such cases, in which the subclass structure is additive (involves no class multiplication). However, it is also possible to apply a similar form of inference in the case of classes that are partitioned multiplicatively, and I refer to this inference as general vicariance. Both forms of inference were assessed in the present study. Therefore, a broader theoretical issue addressed by this research is that of whether additive and multiplicative operations on classes develop simultaneously or

asynchronously. Logically, additive vicariance is a special case of the multiplicative version. I hypothesized that applying vicariance in the case of additive subclasses would be achieved earlier than applying it in the general case of multiplicative subclasses.

Procedure and Subjects

Concrete materials were used to illustrate both types of vicariance problems and to enhance children's comprehension of the questions. Three sets of materials were used, each having a subclass structure that could be construed as either additive or multiplicative.

For instance, one set of materials consisted of two types of plastic animals (dinosaurs and farm animals) intersecting with three colors (red, green, and yellow). Additive vicariance questions ignored this multiplicative structure, referring to the objects only by color. General vicariance questions emphasized the multiplicative structure, referring to objects both by color and animal type. A second set consisted of pictures of apples, plums, and leaves (two colors of each); the third consisted of circles, squares, triangles, and diamonds (four colors of each shape).

Piaget's formulation of vicariance was translated into a logically equivalent form that could be expressed more simply. In the original version, vicariance involves a relation between two subclasses and their complements. For example, using the present materials, "the reds plus the not-reds are equal to the greens plus the non-greens." The translated version involves only a relation between complements and the whole: "The not-reds plus the not-greens are equal to the whole." Questions based on this formulation were further simplified by stating

them in a sequence of three steps. The following example of an additive vicariance question illustrates the method of presentation: Children were asked to pretend that, "(1) first we take away the things that aren't red. (2) Then we also take away the things that aren't green. (3) Is anything left?" Notice that for additive vicariance problems, which refer to disjoint subclasses of colors, the answer to such a question is "no, nothing's left." That is, the not-reds and the not-greens are equal to the whole.

General vicariance questions were stated in the same manner, but referred to intersecting subclasses. For example, "(1) First we take away the things that aren't red. (2) Then we also take away the things that aren't farm animals. (3) Is anything left?" Notice that the answer to such questions is "yes, something is left" (in this case, red farm animals).

In order to further control for effects of language comprehension, subjects were asked a number of preliminary questions about the materials before being given the vicariance questions. These preliminary questions focused on verbal negation; for example, "show me the things that aren't farm animals." The children had to achieve a criterion of success on this series of questions before proceeding to vicariance questions. In addition, the children were required to demonstrate verbal understanding of vicariance questions, by repeating them correctly before answering.

Seventy-two children were tested individually on four additive and three general vicariance problems (i.e., a second additive problem was administered on one task). The subjects ranged in age from 5 years 0 months to 7 years 10 months. Children's answers were probed for

justifications and for certainty. Testing was recorded on videotape. Responses were scored on a 6-point scale, ranging from (Level 0) "Incorrect, makes wild guesses;" to (Level 5) "Immediate solution with certainty, supported by justifications." (See table 1 for a summary of coding criteria).

Insert table 1 about here

Results

First, let's consider the general developmental trends in performance on these tasks. Table 2 shows the percentages of additive vicariance scores obtained by children at different ages. To summarize these data, the majority of the younger subjects score between levels 0 to 3, whereas most of the older subjects score between levels 2 to 5; by the age of 7 years, more than half the children achieve scores of 4-5, on at least one problem. The age trends for multiplicative vicariance are shown in table 3. Performance on all seven problems is significantly associated with age, as measured by Kendall's tau.

Insert tables 2 & 3 about here

Tables 2 and 3 also show that general vicariance scores lag behind additive vicariance scores, as expected. Subjects' mean scores on the additive problems are significantly higher than their mean scores on the

general problems, $t(70) = 7.28$, $p < .001$. This developmental asynchrony was examined in greater detail by tabulating the differences between children's mean scores for additive and general vicariance. This tabulation is presented in table 4.

Insert table 4 about here

As table 4 indicates, all except two of the children obtain additive vicariance scores equal to or higher than their general vicariance scores. This indicates that the asynchrony between the two types of inference is systematic; that is, acquisition of general vicariance appears to presuppose acquisition of additive vicariance.

Two significant differences are found in the nature of errors produced at different developmental levels. As is described in table 1, children scoring at levels 0-1 produce only incorrect answers, whereas those at levels 2-3 give some incorrect and some correct answers. However, children scoring in the 0-1 range make proportionally more errors of complementation than those scoring in the 2-3 range. That is, they are more likely to treat phrases of the form "things that aren't X" as if they refer to things that are X, in spite of the fact that they verbalize such phrases correctly when repeating them to the interviewer. This effect holds for both general ($t(57) = 3.67$, $p < .001$), and additive ($t(47) = 3.71$, $p < .001$), vicariance problems.

Secondly, on general vicariance problems, children with low overall scores make more errors of ignoring the multiplicative subclasses (i.e., they treat intersecting subclasses as if they are disjoint); whereas,

children who score in the middle range (levels 2-3) make more errors in which they take account of the multiplicative subclasses although still reaching the wrong conclusion, ($t(57) = 2.63$, $p < .01$),

Conclusions

These findings support Piaget's interpretation of additive class vicariance as a distinct logical acquisition of the concrete operational period. Performance on these problems shows progressive development across the age range of 5 to 7 years. The younger children have considerable difficulty understanding the class complementation involved. By age 7, children have fewer difficulties with complementation, and most manifest operational vicariance inferences (i.e., obtain scores of 4 or 5) on at least some problems.

Clearly, though, children do not achieve consistent success on these problems even at age 7, and this is especially true for general vicariance problems. Formally, both types of problem are instances of DeMorgan's Law for class logic, that the union of complements is equal to the complement of the intersection (i.e., A' + B' = (AB)'). In the additive case of disjoint subclasses, AB is null (therefore (AB)' is the whole); in the general case AB is not null (therefore (AB)' is not the whole). A possible interpretation of the developmental lag between the two problems is that the additive version can be solved by finding the union of complements (A' + B') in a sequential manner; that is, by performing each complementation separately, then combining the results. Whereas, the general version is solved by finding the complement of intersections ((AB)'), and this requires simultaneously performing both intersection and complementation operations. In other words,

multiplicative vicariance inferences may entail a higher degree of coordination between separate cognitive operations than additive inferences.

Therefore, the findings also shed light on the acquisition of computing the union of complements, and the complement of intersections, during the early concrete operational period. Problems that require computing the union of complements are solved earlier than those that require finding the complement of an intersection. Furthermore, competence with the former appears to be a precondition for competence with the latter.

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Table 1

Summary of Coding Criteria for Vicariance Problems

0.	Incorrect answer or no answer. Answers are wild guesses rather than a product of reasoning.
1.	Incorrect answer, reflecting mistakes of reasoning rather than guessing (supports response with some explanation).
2.	Incorrect answer after first vacillating between right and wrong solutions.
3.	Correct answer by groping, after first vacillating between correct and incorrect solutions.
4.	Correct solution without initial difficulty. Response shows certainty, but is not supported by explanation.
5.	Correct solution immediately, with certainty, and supported by accurate explanation.

Table 2

Percentages of Additive Vicariance Scores by Age

Problem and Score	Age Group				
	5:0-5:5	5:6-5:11	6:0-6:5	6:6-6:11	7:0-7:10
	n = 10	n = 15	n = 17	n = 18	n = 12
Problem 1					
Level 4-5	0	20	24	33	25
Level 2-3	50	40	35	28	50
Level 0-1	50	40	41	39	25
Problem 2					
Level 4-5	0	20	41	45	25
Level 2-3	30	40	12	33	58
Level 0-1	70	40	47	22	17
Problem 3					
Level 4-5	0	34	41	50	58
Level 2-3	70	40	47	39	33
Level 0-1	30	27	12	11	8
Problem 4					
Level 4-5	10	40	53	50	59
Level 2-3	50	33	41	28	33
Level 0-1	40	27	6	23	8

Note. Table has been abbreviated by combining levels 0-1, 2-3, 4-5.

Table 3

Percentages of General Vicariance Scores by Age

Problem and Score	Age Group				
	5:0-5:5	5:6-5:11	6:0-6:5	6:6-6:11	7:0-7:10
	n = 10	n = 15	n = 17	n = 18	n = 12
Problem 1					
Level 4-5	0	14	12	11	8
Level 2-3	11	7	17	22	25
Level 0-1	89	80	71	67	67
Problem 2					
Level 4-5	0	0	12	28	16
Level 2-3	10	20	12	22	42
Level 0-1	90	80	77	50	41
Problem 3					
Level 4-5	0	20	12	33	42
Level 2-3	50	33	53	28	33
Level 0-1	50	46	35	39	25

Note. Table has been abbreviated by combining levels 0-1, 2-3, 4-5.

Table 4

Frequency Tabulation of Difference Scores Representing Synchrony/Asynchrony of Additive and General Vicariance Performance

Difference Between Scores	Frequency
General 5 levels higher than Additive	0
General 4 levels higher than Additive	0
General 3 levels higher than Additive	0
General 2 levels higher than Additive	0
General 1 level higher than Additive	2
General equal to Additive	28
General 1 level lower than Additive	26
General 2 levels lower than Additive	9
General 3 levels lower than Additive	2
General 4 levels lower than Additive	3
General 5 levels lower than Additive	0

Note. Additive score is the average of the four additive vicariance problems; general score is the average of the three general vicariance problems. The frequency distribution differs significantly from that of the null hypothesis of no difference between scores, $Z = 5.68$, $p < .001$ (Modified Wilcoxon test).